Stat 230 - Fall 2010
Exam #1
October 8-11, 2010

NAME {1 pts.}: Key

SHOW ALL WORK ON PROBLEMS REQUIRING CALCULATIONS!!

1. {4 pts.} Suppose we are interested in estimating a parameter θ. Four different estimators are used ($\hat{\theta}_A$, $\hat{\theta}_B$, $\hat{\theta}_C$, and $\hat{\theta}_D$), with each estimate repeated five times. The 5 estimates from each of the estimation methods are listed below.

   - $\hat{\theta}_A$: 8.3, 9.1, 11.2, 13.7, 20.8 ← mean too high; high var = low rel.
   - $\hat{\theta}_B$: 10.7, 11.1, 12.8, 13.8, 13.8 ← " " " ; low var = high rel.
   - $\hat{\theta}_C$: 7.9, 8.5, 9.3, 10.6, 11.8 ← mean is about 10; low var = high rel
   - $\hat{\theta}_D$: 2.0, 7.9, 11.6, 14.4, 15.1 ← " " " " ; high var = low rel

   If the actual value of θ is 10, match the estimator on the left with the best description on the right (each answer is used only once).

   $\hat{\theta}_A$: c (a) low bias & low reliability

   $\hat{\theta}_B$: d (b) low bias & high reliability

   $\hat{\theta}_C$: b (c) high bias & low reliability

   $\hat{\theta}_D$: a (d) high bias & high reliability

2. {3 pts.} Fill in the blanks about the approach to experimental design discussed in class. (One word in each blank is sufficient.) "max-min-con" principle:

   Seek to maximize *the systematic variation of factors of interest and to* minimize *error variance by* controlling *the systematic variation of factors that are not of interest.*

3. {6 pts.} What are the two main reasons we use ***randomization*** when assigning subjects to treatments? (Expecting less than a dozen words per reason.)

   (*i*) protect against bias (reduce bias)

   (*ii*) allows us to use probability and sampling distributions when analyzing the data

4. A gerontologist is interested in studying the effect of: (i) different types of music and (ii) caffeine on the heart rate of human subjects between 80 and 89 years of age. Subjects were randomly assigned (by flip of a fair coin) to either the caffeine group or the non-caffeine group. Each subject drank either a can of Red Bull Energy Drink or an equally-large amount of water and then waited 15 minutes. Then, for each subject, the researcher played hard rock music at 70 decibels (as loud as a vacuum cleaner) for 5 minutes, and then measured the subject's heart rate in beats per minute. Next, each subject listened to classical music at 70 decibels for 5 minutes and then had their heart rate measured. Next, each subject then listened to children's music (the Barney and Friends™ soundtrack) at 70 decibels for 5 minutes and then had their heart rate measured. Finally, each subject sat in silence for 5 minutes and then had their heart rate measured.

   - {2 pts.} What is the response variable?

     heart rate

   - {2 pts.} What are the experimental factors, and what are the levels of each factor?

     (i) types of music (factor levels are: hard rock, classical, children's, & silence)

     (ii) caffeine status (factor levels are: Red Bull & water)

   - {3 pts.} Was blocking used in this study? If so, what was the block? If not, what is a variable that could have been used as a blocking variable to increase the precision of the results?

     Yes. The block is the octogenarian (human) subject.

   - {3 pts.} What is the experimental unit for the comparison of music types?

     The subject during each time period of the study

     four different times

   - {3 pts.} Comment on the strengths/weaknesses of the randomization used in this experiment.

     1 pt → • Randomization for caffeine group is fine, but...

     2 pts → • We should have randomized the order of the 4 treatments given to each subject.

5. {2 pts.} TRUE or FALSE (Circle one) Blocking is a good idea if you can sort your experimental material into groups of units that are similar within groups and different across groups with respect to the likely values for the response.

6. {6 pts.} An experiment is run to evaluate the impact of sleep and diet on cross-country race times. Twelve high school runners are selected that have roughly equivalent average run times for the previous month. The 12 runners are randomly assigned to one of two sleep regimens (6 or 8 hours per night) and one of two diets (high carb or high protein). The runners abide by the sleep/diet assignment for one week and then times are recorded for a 5K at the end of the week. Times (in minutes) for the twelve runners in the study are as follows:

	6 hours sleep	8 hours sleep
High carb	19.3 21.5 18.2	19.2 18.6 18.4
High protein	18.8 19.5 22.2	19.8 17.9 29.5

Calculate the two means that would be appropriate for assessing whether or not there exists an interaction between sleep regimen and diet.

mean of ☐ — mean of ○

21.03 — 19.45

7. {6 pts.} Bags of "Tarties" candies are packaged in bags with a mean weight of 2.01 ounces and a standard deviation of 0.03 ounces. If a case of "Tarties" contains 50 randomly selected bags from the factory, what is the probability that the mean weight of the "Tarties" bags in a case would be less than 2 ounces? Carefully justify your answer—giving the correct number only merits you partial credit. *[Note: If the problem cannot be solved given the stated information, write "Cannot be solved" and explain why the problem cannot be solved, and explain what (if any) information would be needed to solve it.]*

2 pts → Because n is large (50), the sampling distribution of $\bar{X}$ is normal.

$$\Pr\{\bar{X} < 2.0\} = \Pr\left\{Z < \frac{2.0 - 2.01}{.03/\sqrt{50}}\right\}$$

$$= \Pr\{Z < -2.35\}$$

4 pts → $= \boxed{.0094}$

8 ~~8~~. An experiment is run to evaluate whether tomatoes stored at room temperature have better flavor scores than those that have been refrigerated. A sample of 14 tomatoes is obtained, with 7 randomly assigned to a dark container at room temperature and 7 randomly assigned to a dark refrigerator. The flavor scores are as follows:

	Tomato #	1	2	3	4	5	6	7	Mean	Standard Deviation
Grp 1:	Room Temp	80	68	86	74	87	80	81	79.43	6.63
Grp 2:	Fridge	63	70	74	70	75	85	71	72.57	6.70
	Difference	17	-2	12	4	12	-5	10	6.86	8.09

Complete the test of hypotheses below using $\alpha = 0.05$.

STATE: Do tomatoes stored at room temperature have better flavor scores than those that have been refrigerated?

PLAN {2 pts.}: [State the hypotheses] making sure that all symbols are clearly defined]

OR

$H_0: \mu_1 = \mu_2 \rightarrow$ H_0: $\mu_1 - \mu_2 = 0$ where μ_1 is mean flavor for room temp and μ_2 " " " " fridge

$H_a: \mu_1 > \mu_2 \rightarrow$ H_A: $\mu_1 - \mu_2 > 0$

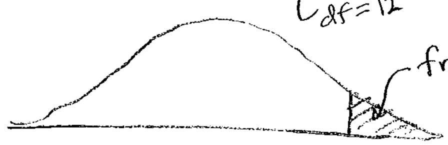
SOLVE {7 pts.}: [Calculate the appropriate test statistic and p-value.]

Two-sample (no pairing involved).

$$s_{pe}^2 = \frac{(6)(6.63^2) + (6)(6.70^2)}{12} = 44.423$$

$$t = \frac{79.43 - 72.57}{\sqrt{\left(\frac{1}{7} + \frac{1}{7}\right) 44.423}} = \frac{6.86}{3.5626} = \boxed{1.926}$$

$t_{df=12}$



1.926

from t-table this is between .025 and .05

$$\boxed{.025 < p\text{-value} < .05}$$

CONCLUDE {2 pts.}: [Draw the appropriate conclusion.]

Reference all four pieces (can combine (3) and (4))

Because our (1) p-value is between .025 and .05, which is (2) less than $\alpha = 0.05$, we (3) reject H_0 and conclude that the (4) mean flavor score for room temperature tomatoes is greater than the mean for refrigerated tomatoes.

9. An owner of an "ice cream truck" business is interested in whether the type of music broadcast by the truck's speakers will affect the sales of that truck. The owner has two trucks and randomly chooses 9 days to run an experiment. On each selected day, he randomly assigns one truck to play traditional ice-cream-truck jingles, and assigns the other truck to play Beyonce's "Single Ladies." The daily sales (in \$) for each truck are recorded below.

Day #	1	2	3	4	5	6	7	8	9	Mean	Standard Deviation
Traditional jingle truck	175	192	184	486	97	358	229	306	179	245.11	118.59
"Single Ladies" truck	115	307	306	628	154	355	309	242	219	292.78	148.23
Difference	60	-115	-122	-142	-57	3	-80	64	-40	-47.67	76.47

Complete the test of hypotheses below using $\alpha = 0.05$.

STATE: Is there a difference in mean daily ice cream sales when Beyonce's "Single Ladies" is played in place of traditional ice-cream-truck jingles?

PLAN {2 pts.}: [State the hypotheses, making sure that all symbols are clearly defined]

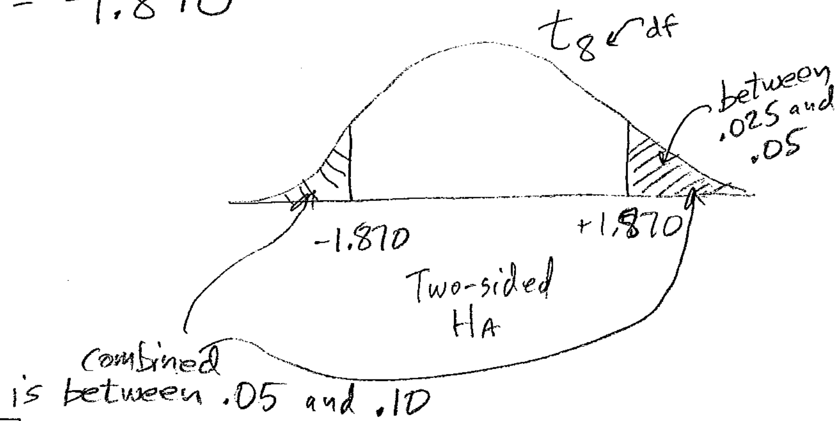
H_0: $\mu_d = 0$ Where μ_d is the mean of the daily differences in sales

H_A: $\mu_d \neq 0$

SOLVE {7 pts.}: [Calculate the appropriate test statistic and p-value.]

Paired sample since we randomly assign treatments within each day.

$$t = \frac{-47.67}{76.47/\sqrt{9}} = -1.870$$



$\boxed{.05 < p\text{-value} < .10}$

CONCLUDE {2 pts.}: [Draw the appropriate conclusion.]

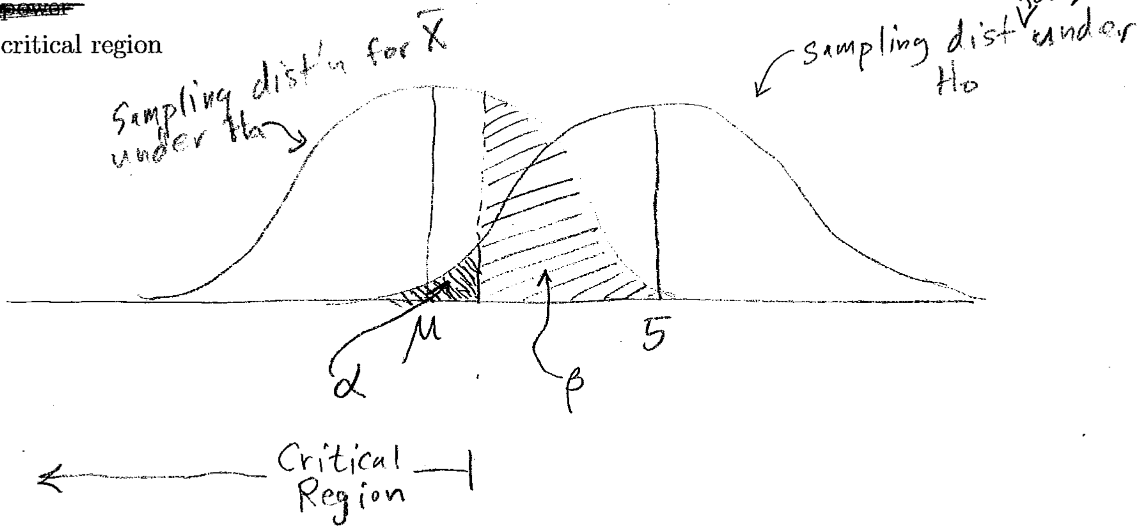
Because our p-value is between .05 and .10, which is greater than $\alpha = 0.05$, we have insufficient evidence to reject the hypothesis that the mean difference is sales for the two truck songs is zero.

OR: insufficient evidence to conclude the mean sales are different for the two truck songs.

10. Consider the hypotheses $H_0 : \mu = 5$ vs. $H_A : \mu < 5$. A sample of size n will be drawn and the sample mean $\bar{x}$ will used for the hypothesis test.

(a) {~~4~~ 7 pts.} Draw a picture with the following components clearly labeled:

- the sampling distribution of $\bar{x}$ under the null hypothesis
- the sampling distribution of $\bar{x}$ under the alternative hypothesis (centered at μ)
- α
- β
- ~~power~~
- critical region



(b) 2 pts ~~{1 pt.}~~ What happens to the power of the test as μ approaches (increases towards) 5?

power will approach the value of α

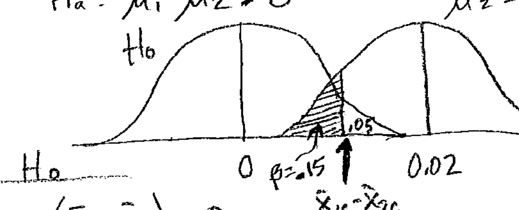
11 ~~8~~. {8 pts.} A researcher studying ankle injuries is interested in characterizing inflammation associated with minor ankle twists. Specifically, she wants to see if twist-related inflammation when wearing flip-flops is greater than twist-related inflammation when wearing traditional shoes with laces. Inflammation values are relative increases in certain foot measurements, and are expected to be in the neighborhood of 1.0 to 1.5 with a standard deviation of 0.09. The researcher intends to use $\alpha = 0.05$ for the hypothesis test. If she would like to have 85% power for detecting a difference of 0.02 in mean inflammation values, how many subjects will she need to recruit for each of the two groups (flip-flop group and traditional shoe group)? (Show all work.)

$$n = \frac{2(.09^2)(\overset{+1.645}{Z_{.05}} + \overset{+1.04}{Z_{.15}})^2}{(.02)^2} = 291.97 \approx 292 \text{ subjects per group}$$

OR

$H_0: \mu_1 - \mu_2 = 0$
$H_a: \mu_1 - \mu_2 > 0$

μ_1 = flip-flop mean
μ_2 = traditional mean



Ho

$$\underset{+1.645}{Z_{.05}} = \frac{(\bar{X}_{1c} - \bar{X}_{2c}) - 0}{\sqrt{2\frac{\sigma^2}{n}}}$$

$$(\bar{X}_{1c} - \bar{X}_{2c}) = 1.645\sqrt{2(.09)^2/n}$$

Ha

$$\underset{-1.04}{Z_{.85}} = \frac{(\bar{X}_{1c} - \bar{X}_{2c}) - (0.02)}{\sqrt{2\sigma^2/n}}$$

$$(\bar{X}_{1c} - \bar{X}_{2c}) = -1.04\sqrt{2(.09)^2/n} + .02$$

set these equal

$$1.645\sqrt{2(.09)^2/n} = -1.04\sqrt{2(.09)^2/n} + .02$$

$$(1.645 + 1.04)\sqrt{2(.09)^2/n} = .02$$

$$(1.645 + 1.04)^2(2)(.09)^2/n = .02^2$$

$$n = \frac{2(.09)^2(1.645 + 1.04)^2}{.02^2}$$

$$= 291.97 \approx 292 \text{ subjects per group}$$